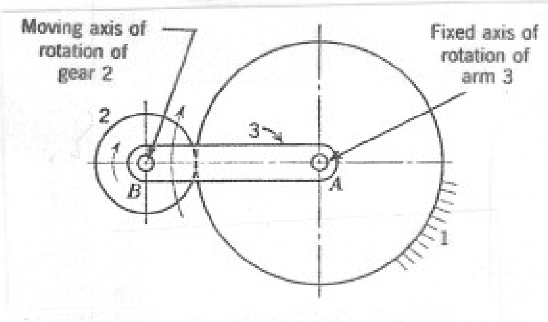
Tabular Method

The tabular method for planetary gear train analysis is based on the following equation:

$$\omega_{gear} = \omega_{arm} + \omega_{gear/arm}$$

- Consider the following planetary gear train with the sun gear fixed.

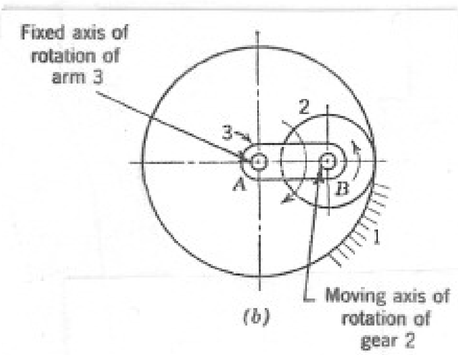


Case	Gear 1	Gear 2	Arm
Arm Velocity	ω_{arm}	ω_{arm}	ω_{arm}
Velocity wrt. Arm	x	$-x \frac{N_1}{N_2}$	0
ω_{Total}	$x + \omega_{arm}$	$\omega_{arm} - x \frac{N_1}{N_2}$	ω_{arm}

If gear 1 is fixed $\omega_1 = x + \omega_{arm} = 0 \quad \therefore \quad x = -\omega_{arm}$

So, $\omega_1 = 0 \qquad \omega_2 = \omega_{arm}\left(1 + \frac{N_1}{N_2}\right)$

- Now let's consider the following planetary gear train with the ring gear being gear 1.



Case	Gear 1	Gear 2	Arm
Arm Velocity	ω_{arm}	ω_{arm}	ω_{arm}
Velocity wrt. Arm	x	$x \frac{N_1}{N_2}$	0
ω_{Total}	$\omega_{arm} + x$	$\omega_{arm} + x \frac{N_1}{N_2}$	ω_{arm}

If gear 1 is fixed: $\omega_1 = \omega_{arm} + x = 0 \rightarrow x = -\omega_{arm}$

So $\omega_1 = 0 \qquad \omega_2 = \omega_{arm}\left(1 - \frac{N_1}{N_2}\right)$

$$\frac{\omega_2}{\omega_{arm}} = \left(1 - \frac{N_1}{N_2}\right) \qquad \text{If } N_1 = 101 \text{ and } N_2 = 100$$

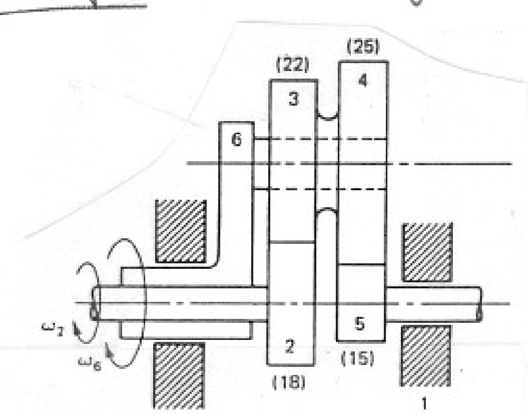
$$\omega_2 = -\frac{1}{100}\,\omega_{arm} \qquad \text{This is a very large gear ratio!}$$

If gear 1 is free and gear 2 is fixed

$$\omega_2 = \omega_{arm} + x\frac{N_1}{N_2} = 0 \rightarrow x = -\frac{N_2}{N_1}\,\omega_{arm}$$

$$\text{so} \quad \omega_2 = 0 \qquad \omega_1 = \omega_{arm}\left(1 - \frac{N_2}{N_1}\right)$$

<u>Example</u> previously solved on p. 12-2 using the Formula Method



Determine the angular velocity of gear 5

Given: $\omega_2 = 50$ r/s

$\omega_6 = 75$ r/s

both counterclockwise when viewed from the right

Case	Gear 2	Gear 3	Gear 4	Gear 5	Arm
Arm Velocity	75	75	75	75	75
Velocity wrt. Arm	x	$-x\frac{N_2}{N_3}$	$-x\frac{N_2}{N_3}$	$x\frac{N_2 N_4}{N_3 N_5}$	0
ω_{Total}	$x + 75$	$75 - x\frac{N_2}{N_3}$	$75 - x\frac{N_2}{N_3}$	$75 + x\frac{N_2 N_4}{N_3 N_5}$	75

$$\text{since} \quad \omega_2 = 50\text{ r/s} = x + 75 \rightarrow x = -25\text{ r/s}$$

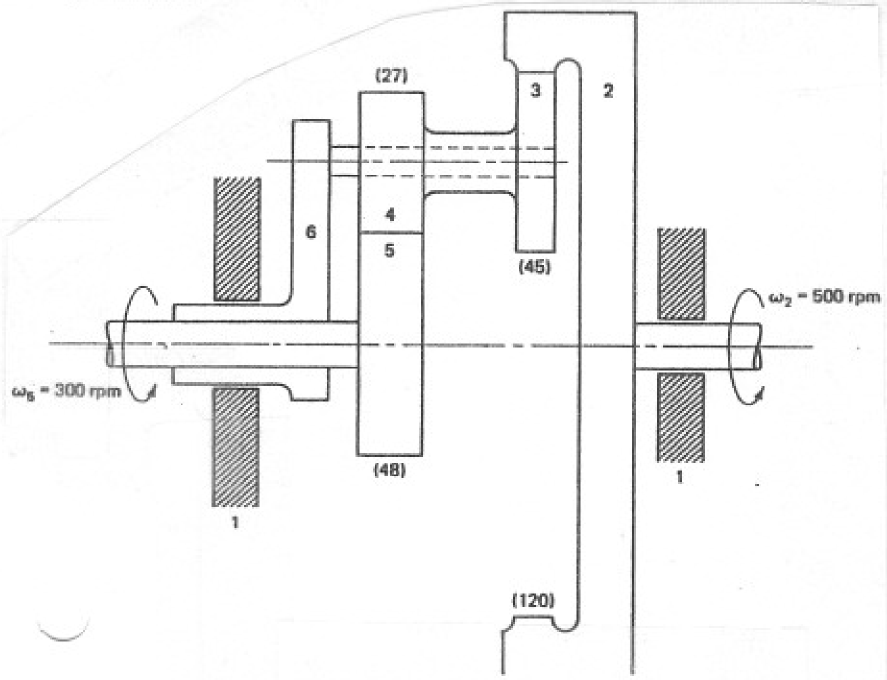
$$\omega_3 = 75 - (-25)\left(\frac{18}{22}\right) = 95.45\text{ r/s}$$

$$\omega_4 = \omega_3 = 95.45 \text{ r/s}$$

$$\omega_5 = 75 + (-25)\frac{(18)(25)}{(22)(15)} = \boxed{40.91 \text{ r/s} = \omega_5}$$

This is the same result on p. 12-3

Example previously solved on p 12-3 using the Formula Method



Two-degree-of-freedom planetary gear train; 2 and 5 are inputs, 6 is output

For the planetary gear system shown, gear 2 and gear 5 are the inputs rotating counter-clockwise from the right

$\omega_2 = 500$ rpm

$\omega_5 = 300$ rpm

Find $\omega_6 = \omega_{arm}$

Case	Gear 2	Gear 3	Gear 4	Gear 5	Arm
Arm Velocity	ω_{arm}	ω_{arm}	ω_{arm}	ω_{arm}	ω_{arm}
Velocity wrt. Arm	x	$x\frac{N_2}{N_3}$	$x\frac{N_2}{N_3}$	$-x\frac{N_2N_4}{N_3N_5}$	0
ω_{Total}	$\omega_{arm}+x$	$\omega_{arm}+x\frac{N_2}{N_3}$	$\omega_{arm}+x\frac{N_2}{N_3}$	$\omega_{arm}-x\frac{N_2N_4}{N_3N_5}$	ω_{arm}

$$\omega_2 = \omega_{arm} + x = 500 \rightarrow -x = \omega_{arm} - 500$$

$$\omega_5 = \omega_{arm} - x\frac{N_2N_4}{N_3N_5} = 300 \rightarrow$$

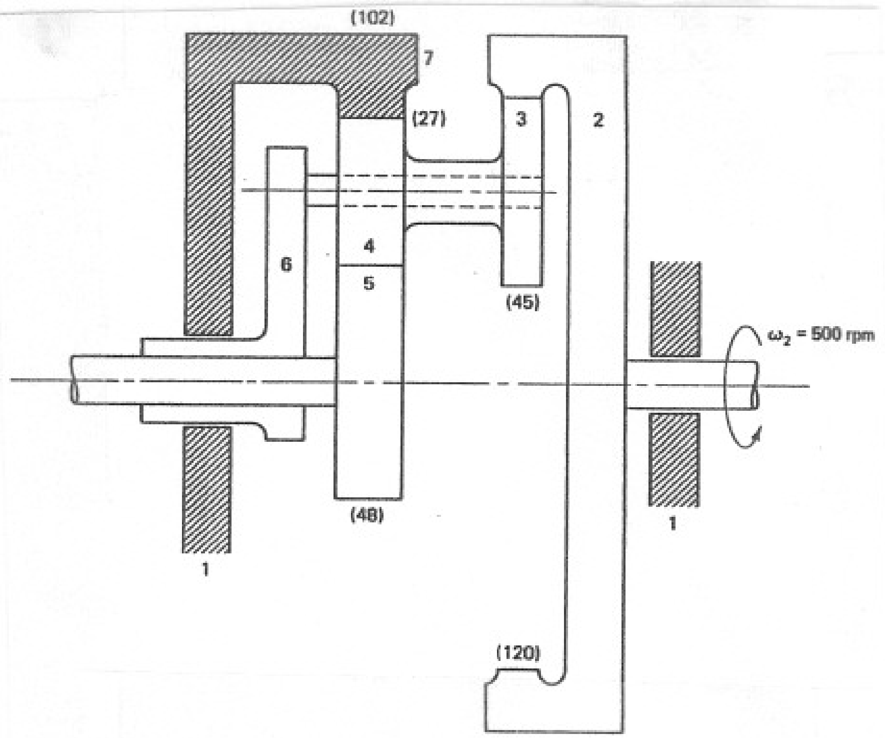
$$\rightarrow \quad \omega_{arm} + (\omega_{arm} - 500)\frac{N_2 N_4}{N_3 N_5} = 300$$

$$\omega_{arm}\left(1 + \frac{N_2 N_4}{N_3 N_5}\right) = 300 + 500\,\frac{N_2 N_4}{N_3 N_5}$$

$$\omega_{arm} = \frac{300 + 500\left(\frac{N_2 N_4}{N_3 N_5}\right)}{1 + \frac{N_2 N_4}{N_3 N_5}} = \frac{300 + 500\left(\frac{(120)(27)}{(45)(48)}\right)}{\left(1 + \frac{(120)(27)}{(45)(48)}\right)} = 420 \text{ rpm}$$

$$\boxed{\omega_{arm} = 420 \text{ rpm}}$$

Example previously solved on p. 12-4 using the Formula Method



Gear 2 is the input rotating at 500 rpm ccw as viewed from the right

Find ω_5

Example cont.

Case	Gear 2	Gear 3	Gear 4	Gear 5	Gear 7	Arm
Arm Velocity	ω_{arm}	ω_{arm}	ω_{arm}	ω_{arm}	ω_{arm}	ω_{arm}
Velocity wrt. Arm	x	$x\frac{N_2}{N_3}$	$x\frac{N_2}{N_3}$	$-x\frac{N_2N_4}{N_3N_5}$	$x\frac{N_2N_4}{N_3N_7}$	0
ω_{TOTAL}	$\omega_{arm}+x$	$\omega_{arm}+x\frac{N_2}{N_3}$	$\omega_{arm}+x\frac{N_2}{N_3}$	$\omega_{arm}-x\frac{N_2N_4}{N_3N_5}$	$\omega_{arm}+x\frac{N_2N_4}{N_3N_7}$	ω_{arm}

$$\omega_2 = \omega_{arm} + x = 500$$

$$\omega_7 = \omega_{arm} + x\frac{N_2N_4}{N_3N_7} = 0 \rightarrow \omega_{arm} = -x\frac{N_2N_4}{N_3N_7}$$

Combining these two equations

$$500 = x\left(1 - \frac{N_2N_4}{N_3N_7}\right) \rightarrow x = \frac{500}{1-\frac{(120)(27)}{(45)(102)}} = 1700 \text{ rpm}$$

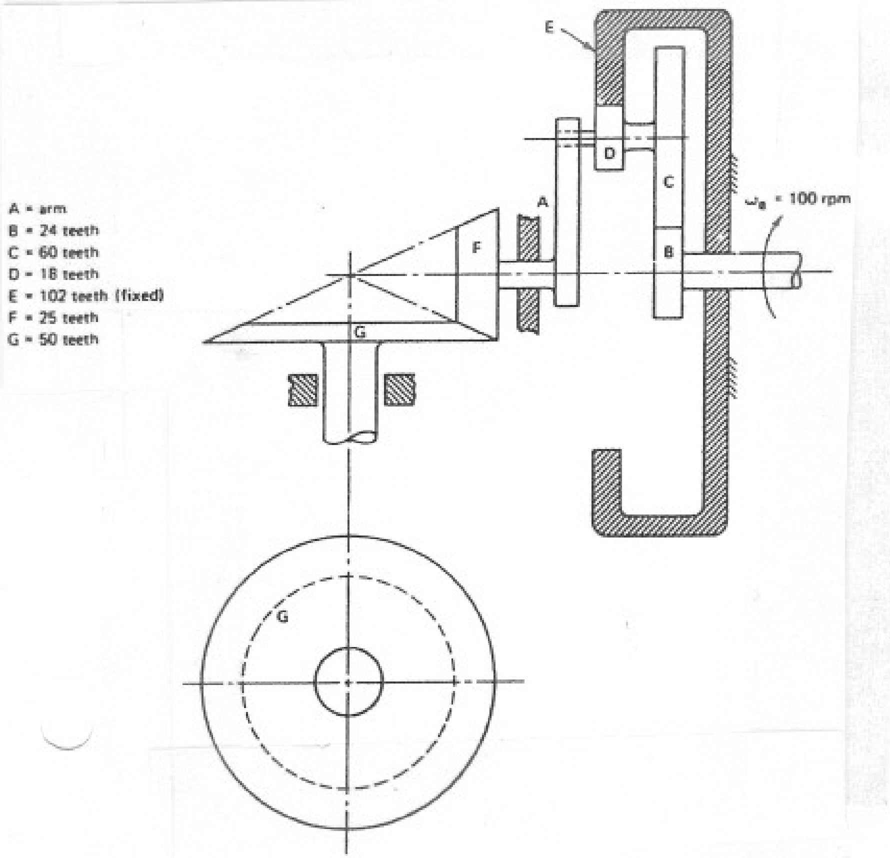
$$\omega_{arm} = 500 - 1700 = -1200 \text{ rpm}$$

$$\therefore \omega_5 = -1200 - 1700\frac{(120)(27)}{(45)(48)} = \boxed{-3750 \text{ rpm} = \omega_5}$$

Example

The sun gear rotates at 100 rpm CW as viewed from the right. Determine the angular velocity and direction of gear G as viewed from the bottom. 13-6

(previously solved using the Formula method on p 12-5)



Case					
Arm Velocity					
Velocity wrt. Arm					
Total					